Scale-Adaptive Simulation (SAS) Turbulence Modeling

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Unsteady RANS Based Models

- **URANS (Unsteady Reynolds averaged Navier Stokes) Methods**
  - URANS gives unphysical single mode unsteady behavior
  - Some improvement relative to steady state (RANS) but often not sufficient to capture main effects
  - Reduction of time step and refinement of mesh do not benefit the simulation

- **SAS (Scale-Adaptive Simulation) Method**
  - Extends URANS to many technical flows
  - Provides “LES”-content in unsteady regions
  - Produces information on turbulent spectrum
  - Can be used as basis for acoustics simulations
Assumptions Two-Equation Models

- Largest eddies are most effective in mixing
- Two scales are minimum for statistical description of large turbulence scales
- Two model equations of independent variables define the two scales
  - Equation for turbulent kinetic energy is representing the large scale turbulent energy
  - Second equation ($\varepsilon$, $\omega$, $kL$) to close the system
  - Each equation defines one independent scale
  - Both $\varepsilon$- and $\omega$-equations describe the smallest (dissipate) eddies, whereas two-equation models describe the largest scales
  - Rotta developed an exact transport equation for the large turbulent length scales. This is a much better basis for a term-by-term modelling approach
Classical Derivation 2 Equation Models

- **The k-equation:**
  - Can be derived exactly from the Navier-Stokes equations
  - Term-by-term modelling
- **The $\varepsilon$- ($\omega$-) equation:**
  - Exact equation for smallest (dissipation) scales
  - Model for large scales not based on exact equation
  - Modelled in analogy to $k$-equation and dimensional analysis
  - Danger that not all effects are included

\[
\frac{\partial (\rho k)}{\partial t} + \frac{\partial (\rho U_j k)}{\partial x_j} = P_k - c_{\mu} \rho k \omega + \frac{\partial}{\partial x_j} \left( \frac{\mu_t}{\sigma_k} \frac{\partial k}{\partial x_j} \right)
\]

\[
\frac{\partial (\rho \omega)}{\partial t} + \frac{\partial (\rho U_j \omega)}{\partial x_j} = \alpha \left( \frac{\omega}{k} \right) P_k - \beta \left( \frac{\omega}{k} \right) \rho (k \omega) + \frac{\partial}{\partial x_j} \left( \frac{\mu_t}{\sigma_\omega} \frac{\partial \omega}{\partial x_j} \right)
\]

\[
\mu_t = \rho \frac{k}{\omega}
\]
Source Terms Equilibrium – k-ω Model

Only one Scale in Sources (S~1/T)

\[
\begin{align*}
\frac{\partial (\rho k)}{\partial t} + \frac{\partial (\rho U_j k)}{\partial x_j} &= \mu_t \left( S^2 - c_\mu \omega^2 \right) + \frac{\partial}{\partial x_j} \left( \frac{\mu_t}{\sigma_k} \frac{\partial k}{\partial x_j} \right) \\
\frac{\partial (\rho \omega)}{\partial t} + \frac{\partial (\rho U_j \omega)}{\partial x_j} &= \rho \left( c_{\omega_1} S^2 - c_{\omega_2} \omega^2 \right) + \frac{\partial}{\partial x_j} \left( \frac{\mu_t}{\sigma_\omega} \frac{\partial \omega}{\partial x_j} \right)
\end{align*}
\]

Input S → Turbulence Model → Output k

Output ω

One input scale – two output scales?
Source terms do not contain information on two independent scales
Determination of $L$ in $k-\omega$ Model

**k-equation:**

$$\frac{\partial (k)}{\partial t} + \frac{\partial (U_k k)}{\partial x_k} = \frac{k}{\omega} (S^2 - c_\mu \omega^2) + \frac{\partial}{\partial y} \left[ \frac{k}{\omega} \frac{\partial k}{\partial y} \right]$$

- Diffusion term carries information on shear-layer thickness $\delta$
- Turbulent length scale proportional to shear layer thickness
- Finite thickness layer required
- Computed length scale independent of details inside turbulent layer
- No scale-resolution, as $L_t$ always large and dissipative

$$0 = \frac{k}{\omega} (S^2 - c_\mu \omega^2) + c \frac{1}{\delta} \left[ \frac{k}{\omega} \frac{k}{\omega} \right]$$

$$\omega \sim S \quad \text{from } \omega\text{-equation}$$

$$0 = cS^2 + \tilde{c} \frac{k}{\delta^2} \quad k \sim S^2 \delta^2$$

$$L_t \sim \frac{\sqrt{k}}{\omega} \sim \frac{\sqrt{S^2 \delta^2}}{S} \sim \delta$$
Rotta’s Length Scale Equation

• To avoid the problem that the $\varepsilon(\omega)$ equation is an equation for the smallest scales, an equation for the large (integral) scales is needed.

• This requires first a mathematical definition of an integral length scale, $L$.
  – In Rotta’s (1968) approach this definition is based on two-point correlations

• Based on that definition of $L$, an exact transport equation can be derived from the Navier-Stokes equations (the actual equation is based on $kL$)

• This exact equation is then modelled term-by-term

Two-Point Velocity Correlations

Measurement of velocity fluctuations with two probes at two different locations
For small \( r \), all eddies contribute
For large \( r \), only large scales contribute
For \( r > L \), correlation goes to zero
Integral vs. \( r \) proportional to size of large eddies \( L \)

\[
\tilde{R}_{ij} = \frac{u_i'(\bar{x}, t)u_j'(\bar{x} + \vec{r}, t)}{u_i'(\bar{x}, t)u_j'(\bar{x}, t)}
\]
Rotta’s $k-kL$ Model

Integral Length Scale:

- The integral of the correlations provides a quantity, $L$, with dimension ‘length’.
- $L$ is based only on velocity fluctuations and can therefore be described by the Navier-Stokes equations.
- Exact equation for $L$ (or $kL$, ..) can be derived.
- $L$ is a true measure of the size of the largest eddies.
Exact Transport Equation Integral Length-Scale (Rotta)

Exact transport equations for $\Phi = kL$ (boundary layer form):

$$\frac{\partial (\Phi)}{\partial t} + \frac{\partial (U_k \Phi)}{\partial x_k} = -\frac{3}{16} \frac{\partial U (x)}{\partial y} \int R_{21} dr_y - \frac{3}{16} \int \frac{\partial U (x + r_y)}{\partial y} R_{12} dr_y +$$

$$\frac{3}{16} \int \frac{\partial}{\partial r_k} (R_{ik} - R_{i(k)}) dr_y + \nu \frac{3}{8} \int \frac{\partial^2 R_{ii}}{\partial r_k \partial r_k} dr_y -$$

$$\frac{\partial}{\partial y} \left\{ \frac{3}{16} \int \left[ R_{(i2)i} + \frac{1}{\rho} \left( p'v + vp' \right) \right] - \nu \frac{\partial}{\partial y} \left( \Phi \right) \right\} \text{ with } \Phi = kL(x)$$

- Important term:
Expansion of Gradient Function

Important term:

\[
\frac{\partial U( x + r_y )}{\partial y} = \frac{\partial U( x )}{\partial y} + \frac{\partial^2 U( x )}{\partial y^2} r_y + \frac{\partial^3 U( x )}{\partial y^3} \frac{r_y^2}{2} + \ldots
\]

\[
\int \frac{\partial U(x + r_y)}{\partial y} R_{12} dr_y \rightarrow \frac{\partial U(x)}{\partial y} \int R_{12} dr_y + \frac{\partial^2 U(x)}{\partial y^2} \int r_y R_{12} dr_y + \frac{1}{2} \frac{\partial^3 U(x)}{\partial y^3} \int r_y^2 R_{12} dr_y
\]

- Rotta:

\[
\frac{\partial^2 U(x)}{\partial y^2} \int r_y R_{12} dr_y = 0
\]

- Due to symmetry of \( R_{ij} \) with respect to \( r_y \) for homogeneous turbulence
Transport Equation Integral Length-Scale (Rotta)

Transport equations for $kL$:

$$
\frac{\partial (\rho \Phi)}{\partial t} + \frac{\partial (\rho U_k \Phi)}{\partial x_k} = -\mu \nu \left( \zeta L \frac{\partial U_i(x)}{\partial y} + \zeta_3 L^3 \frac{\partial^3 U_i(x)}{\partial y^3} \right) - c_L c \rho \left( \frac{q^2}{2} \right)^{3/2} + \frac{\partial}{\partial y} \left\{ \frac{\mu \nu}{\sigma_\Phi} \frac{\partial}{\partial y} (\Phi) \right\}
$$

- Equation has a natural length scale:

$$
L^2 = \frac{c_L - c}{\zeta_3} \frac{\partial U / \partial y}{\partial^3 U / \partial y^3}
$$

- Problem – 3rd derivative:
  - Non-intuitive
  - Numerically problematic

- If $\zeta_3 = 0$ - No natural length scale
  - No fundamental difference to other scale-equations
Virtual Experiment 1D Flow

\[ \frac{\partial^2 U}{\partial y^2} \int r_y R_{12} dr_y = 0 ? \]

Logarithmic layer \( L_t = \kappa y \)

\[ \tilde{R}_{12} = \frac{u(x)v(x + r_y)}{u(x)v(x)} \]

\[ u(x)v(x) = \text{const.} = \frac{\tau_w}{\rho} \]

\[ \tilde{R}_{12}^I (\bar{r}_y) < \tilde{R}_{12}^II (\bar{r}_y) \]

\[ \tilde{R}_{12}^{III} (\bar{r}_y) = \tilde{R}_{12}^II (\bar{r}_y) \quad \tilde{R}_{12}^{III} (\bar{r}_y) \approx \tilde{R}_{12}^I (-\bar{r}_y) \]

\[ \tilde{R}_{12}^{III} (-\bar{r}_y) < \tilde{R}_{12}^{III} (\bar{r}_y) \]

\( R_{12} \text{ asymmetric} \)

\[ \int r_y R_{12} dr_y \neq 0 \]
New 2-Equation Model (KSKL)

\[
\frac{\partial (k)}{\partial t} + \frac{\partial (U_j k)}{\partial x_j} = \nu_t \frac{c_{\mu}^{3/4}}{L} + \frac{\partial}{\partial x_j} \left( \frac{\nu_t}{\sigma_k} \frac{\partial k}{\partial x_j} \right) - \zeta_1 P_k - \zeta_2 \frac{1}{\kappa^2} L^2 \nu_t \left( \frac{U''}{U'} \right)^2 - \zeta_3 k + \frac{\partial}{\partial y} \left[ \frac{\nu_t}{\sigma_y} \frac{\partial \Phi}{\partial y} \right]
\]

- With:

\[
\Phi = \sqrt{kL}, \quad \nu_t = c_{\mu}^{1/4} \Phi
\]

\[|U'| = \left( \frac{\partial U_i}{\partial x_j} \frac{\partial U_i}{\partial x_j} \right)^{1/2}, \quad |U''| = \sqrt{\frac{\partial^2 U_i}{\partial x_j \partial x_j} \frac{\partial^2 U_i}{\partial x_k \partial x_k}}; \quad L_{vK} = \kappa \left| \frac{U'}{U''} \right|
\]

- v. Karman length-scale as natural length-scale:

\[
L \sim \kappa \left| \frac{\partial U / \partial y}{\partial^2 U / \partial y^2} \right| = L_{vK}
\]
SAS Model Derivation

- Using the exact definition and transport equation of Rotta, we re-formulated the equation for the second turbulence scale.
- We use a term-by-term modelling approach based on the exact equation.
- This results in the inclusion of the second velocity derivative $U''$ in the scale equation
- Based on $U''$ the scale equation is able to adjust to resolved scales in the flow.
- The KSKL model is one variant of the SAS modelling concept, as these terms can also be transformed into other equations ($\varepsilon$- or $\omega$).
Transformation of SAS Terms to SST Model

- Transformation:

\[
\Phi = \frac{1}{c_\mu^{1/4}} \frac{k}{\omega}
\]

\[
\frac{D \omega}{Dt} = \frac{1}{c_\mu^{1/4}} \frac{D}{Dt} \left( \frac{k}{\Phi} \right) = \frac{1}{c_\mu^{1/4}} \left( \frac{1}{\Phi} \frac{Dk}{Dt} - \frac{k}{\Phi^2} \frac{D\Phi}{Dt} \right) = \frac{\omega}{k} \frac{Dk}{Dt} - \frac{\omega}{\Phi} \frac{D\Phi}{Dt}
\]

\[
\frac{\partial \rho \omega}{\partial t} + \frac{\partial U_j \rho \omega}{\partial x_j} = \alpha \rho S^2 - \beta \rho \omega^2 + \frac{\partial}{\partial x_j} \left( \frac{\mu_t}{\sigma_\omega} \frac{\partial \omega}{\partial x_j} \right) + \frac{2 \rho}{\sigma_\phi} \left( \frac{1}{\omega} \frac{\partial k}{\partial x_j} \frac{\partial \omega}{\partial x_j} - \frac{k}{\omega^2} \frac{\partial \omega}{\partial x_j} \frac{\partial \omega}{\partial x_j} \right) + \zeta_2 \kappa \rho S^2 \left( \frac{L}{L_v K} \right)^2
\]

Wilcox Model  BSL (SST) Model  SAS

\[
L_v K = \kappa \left| \frac{\partial U / \partial y}{\partial^2 U / \partial y^2} \right|
\]
2-D Stationary Flows: KSKL - RANS

NACA-4412 airfoil at 14°: trailing edge separation
Eddy growth limited by \( \Lambda vK \).

Eddies grow to infinity.
One Model – Two Modes

RANS Model $L \sim \delta$
SAS $L \sim \lambda$

$$U(y) = U_0 \sin \left( \frac{2\pi \cdot y}{\lambda} \right)$$
SAS Modell - 2D Periodic Hill

Scale-Adaptation based on $\Delta t$

$\Delta t = 0.045 \frac{h}{U_B}$

4× higher $\Delta t$

2× higher $\Delta t$
SAS Modell - 2D Periodic Hill

Time averaged velocity profiles $U$

- LES, Temmerman and Leschziner
- SST-SAS, $\Delta t = 0.045\ U_B/h$
- SST-SAS, $\Delta t = 0.18\ U_B/h$
- 2-D SST-RANS
Fluent-SAS Model
Volvo Bluff Body : Cold Case

SAS-SST

DES-SST

Q = $1 \times 10^6$
VOLVO Cold Case

Time-averaged U-velocity
Test case: Mirror Geometry

EU project DESIDER Testcase

Plate dimensions $L \times W = 2.4 \times 1.6$

Cylinder Diameter : $D = 0.2 \text{ m}$

Rear Face location : $0.9 \text{ m}$

Free stream Velocity: $140 \text{ km/h}$

$Re_D : 520 \, 000$

Mach: $0.11$
Test case: Mesh

Mesh: Box around the Plate & Cylinder
- Height of domain: 10 diameters (D=0.2m)
- Coarse and fine meshes
- wall-normal distance around $1-3 \times 10^{-4}$ m
- obstacle edges resolution: step sizes around $0.02 \times D$ (height) - $0.03 \times D$ (circumf.)

Flow: Air as ideal gas
Validation: Near field SPL

Sensors downstream the mirror

Grid ~ 3 million nodes
Blow-Down Simulation – SAS (SST)

Mesh – 1x10^7 control volumes hybrid unstructured

Scale resolving results:
- SAS and DES show similar flow pattern
- SAS model does not rely on grid spacing
- SAS can be applied to moving meshes with more confidence

Courtesy VW AG Wolfsburg: O. Imberdis, M. Hartmann, H. Bensler, L. Kapitza VOLKSWAGEN AG, Research and Development, Wolfsburg, Germany D. Thevenin University of Magdeburg
## Flow Topology and Mass Flow

### Mass flow Rates

<table>
<thead>
<tr>
<th>Intake Valve</th>
<th>Exp.</th>
<th>RANS</th>
<th>DES</th>
<th>SAS</th>
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<tbody>
<tr>
<td>3 mm</td>
<td>1</td>
<td>0.95</td>
<td>0.985</td>
<td>0.996</td>
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<tr>
<td>9 mm</td>
<td>1</td>
<td>0.988</td>
<td>-</td>
<td>0.99</td>
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</tbody>
</table>

Courtesy VW AG Wolfsburg: O. Imberdis, M. Hartmann, H. Bensler, L. Kapitza
VOLKSWAGEN AG, Research and Development, Wolfsburg, Germany
D. Thevenin University of Magdeburg
Geometry of the Cavity

- D = 4 in
- L = 5 D, W = D
- \( L_x \times L_y \times L_z = 18 \, D \times 17 \, D \times 9 \, D \)
- M = 0.85
- P = 62100 Pa
- T = 266.53 K
- Re = 13.47 \times 10^6
Mesh: 5.8 e 6 Cv – double O-grid
Turbulent structure by $q$-criterion

Eddy viscosity ratio @ $q = -500000$ ($q = \frac{1}{2} (S:S - \Omega:\Omega)$)
Wave propagation by Fluctuating Density

Eddy viscosity ratio @ \( q = -500000 \) \( (q = \frac{1}{2} (S:S - \Omega:\Omega)) \)
k20 – k25

Graphs showing data for k20, k21, k22, k23, k24, and k25. Each graph compares 'Experiment' with 'FLUENT SAS' data across a range of frequencies from 100 Hz to 1000 Hz.

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k26 – k29
Testcase Description – Experimental Test Facility and Data

• The experimental data is provided by the Institute of Aerodynamics and Fluid Mechanics from TUM (not yet released)

• Experiments are performed including a moving belt

Courtesy by TU Munich, Inst. of Aerodynamics
Computational Mesh 2

- 108,034,893 Cells
- Four Refinement Boxes
- MRF-Zones

<table>
<thead>
<tr>
<th>Number of Inflations</th>
<th>First layer height</th>
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<tbody>
<tr>
<td>Car</td>
<td>20</td>
</tr>
<tr>
<td>Road</td>
<td>20</td>
</tr>
</tbody>
</table>

Courtesy by TU Munich, Inst. of Aerodynamics
DrivAir Generic Car Model

- Courtesy Tu Munich
- Currently studied with ANSYS CFD (Fluent and CFX)
- Data not yet public
Overall Summary

• SAS is a second generation URANS model
  – It is derived on URANS arguments
  – It can resolve turbulence structures with LES quality
  – A strong flow instability is required to generate new – resolved turbulence

• Examples
  – Flows past bluff bodies
  – Strongly swirling flows (combustion chamber)
  – Strongly interacting flows (mixing of two jets etc.)

• SAS Model is first and relatively save step into Scale-Resolving Simulations (SRS) modeling
  – Worthwhile to try
  – Alternative Detached Eddy Simulation (DES)